

TRANSMISSION THROUGH AN INTERACTING QUANTUM DOT IN THE COULOMB BLOCKADE REGIME

R. Berkovits¹, U. Sivan²

¹*The Jack and Pearl Resnick Institute of Advanced Technology,
Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel*

²*Solid State Institute and Physics Department, Technion,
Israel Institute of Technology, Haifa, Israel*

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Abstract

The influence of electron-electron (e-e) interactions on the transmission through a quantum dot is investigated numerically for the Coulomb blockade regime. For vanishing magnetic fields, the conductance peak height statistics is found to be independent of the interactions strength. It is identical to the statistics predicted by constant interaction single electron random matrix theory and agrees well with recent experiments. However, in contrast to these random matrix theories, our calculations reproduces the reduced sensitivity to magnetic flux observed in many experiments. The relevant physics is traced to the short range Coulomb correlations providing thus a unified explanation for the transmission statistics as well as for the large conductance peak spacing fluctuations observed in other experiments.

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Since the discovery of the Coulomb blockade phenomenon, most tunneling experiments through a quantum dot were interpreted within the constant interaction model (CI) [1,2]. In that approximation, the ground state energy of a quantum dot populated by N electrons is expressed as $E_N = \frac{e^2 N^2}{2C} + \sum_{i=1}^N \eta_i$ where C is the dots constant (or slowly varying) capacitance, and η_i are the single particle energies. Evidently, in that model only the long range Coulomb interaction is taken into account while the short range correlations are neglected. The fine ground state properties are hence determined by the single particle states which for a disordered or chaotic dot display a random matrix theory (RMT) statistics. Although the CI model is very appealing in its simplicity, it was recently proved wrong in predicting the the distribution of the conductance peak spacings [3–5], as well as the large peak spacing fluctuations found in some of the experiment [3,4]. While the CI model predicts a RMT type of ground state statistics with a characteristic energy scale, Δ (average single particle level spacing), the experiments find a different type of statistics and (at least for the experiments in Ref. [3,4]) considerably larger fluctuations which seem to be independent of Δ . Moreover, the ground state energy turns out to be relatively insensitive to magnetic flux [4] and application of one quantum flux unit, $\phi_0 = hc/e$, through the dot hardly affect it [6,7]. This insensitivity is again in contrast with the CI model since the single particle states, and hence the ground state energy in that model, are expected to fluctuate on a flux scale smaller than one quantum flux unit [8,9].

A point of view similar to the CI one was also taken for the calculation of the Coulomb blockade conductance peak height statistics [10–13]. Since RMT was assumed for the single particle states, a modified Porter-Thomas distribution was obtained for the dimensionless transmission $\alpha \equiv 2\Gamma_L\Gamma_R/\Gamma\langle\Gamma\rangle$

$$\begin{aligned}
P_{B=0}(\alpha) &= \sqrt{\frac{2}{\pi\alpha}} e^{-2\alpha}; \\
P_{B\neq 0}(\alpha) &= 4\alpha[K_0(2\alpha) + K_1(2\alpha)]e^{-2\alpha}.
\end{aligned}
\tag{1}$$

Here, $\Gamma_L(\Gamma_R)$ are the tunneling rates from the left (right) lead, $\Gamma = \Gamma_L + \Gamma_R$, $\langle \dots \rangle$ denotes an average over different peaks or disorder realizations and K_0, K_1 are the modified Bessel

functions.

Subsequent experiments [6,7] reported partial agreement with these calculations and one was therefore facing the following dilemma: while the conductance peak spacing fluctuations can be explained by short range Coulomb correlations [3] (see also recent papers by Koutrakov et al. [14] and Blanter et al. [15]), the conductance peak height statistics roughly agrees with a model that totally neglects these correlations. It is hard to reconcile such two different pieces of physics for two facets of the same phenomenon and in the present manuscript we show indeed that the observed peak height statistics may result from fluctuations in the short range Coulomb correlations rather than the single electron RMT physics (i.e., wave functions with no correlations) utilized in refs. [10–13]. Strong support in favor of the Coulomb correlations type of physics comes from the insensitivity to magnetic flux observed in both experiments [6,7]. The auto correlation function between the height of a given peak at two different values of the magnetic flux, ϕ ,

$$C(\phi, \Delta\phi) \equiv \frac{\langle \delta\alpha(\phi) \delta\alpha(\phi + \Delta\phi) \rangle}{\sqrt{\langle \delta^2\alpha(\phi) \rangle \langle \delta^2\alpha(\phi + \Delta\phi) \rangle}} ; \quad \delta\alpha = \alpha - \langle \alpha \rangle , \quad (2)$$

is found experimentally to decay on flux scales larger than predicted by CI single electron RMT. This reduced sensitivity to flux is clearly manifested in the numerical calculation presented below that take the interactions into account. Curiously, the inclusion of interactions does not change the peak height distribution predicted by single electron RMT and observed by the experiments. We are therefore able to propose a unified explanation for both facets of the Coulomb blockade phenomenon, namely, the conductance peak spacing fluctuations and the peak height distribution. They both may originate from fluctuations in the Coulomb interaction rather than single particle physics [3].

The height g_{max} of a conductance peak is given by

$$g_{max} = \frac{e^2}{h} \left(\frac{\pi}{2k_B T} \right) \langle \Gamma \rangle \alpha. \quad (3)$$

The tunneling rates may be formulated in a tight-binding many particle language as

$$\Gamma_{L(R)} = |t_{L(R)}|^2 \sum_{k,j \in [L(R)]} \langle \Psi_{N+1} | a_{k,j}^\dagger | \Psi_N \rangle|^2, \quad (4)$$

where $t_{L(R)}$ is the barrier transmission which is assumed to depend only weakly on energy, Ψ_N is the N particle ground state wave function in the dot, $a_{k,j}^\dagger$ is the fermionic creation operator at site (k, j) , and summation is performed on sites $k, j \in [L(R)]$ i.e, sites adjacent to the left (right) lead. This is a straight forward adaptation of the definition of $\Gamma_{L(R)}$ given in Ref. [10]. For the non-interacting case, assuming statistically identical independent single channel leads (for example, point contacts [11]), RMT predicts the distribution depicted in Eq. (1) [10].

We calculate the tunneling rates $\Gamma_{L(R)}$ for a system of interacting electrons modeled by a tight-binding Hamiltonian. We choose a 2D cylindrical geometry of circumference L_x and height L_y . This particular geometry is very convenient for the study of the influence of a magnetic flux ϕ threading the cylinder in the \hat{y} direction. A radial magnetic field could also be applied, but for a field equivalent to one quantum flux unit through the system, Landau bands appear. Since this is not the situation in the experiment we prefer to apply a threading flux only. The Hamiltonian is given by:

$$H = \sum_{k,j} \epsilon_{k,j} a_{k,j}^\dagger a_{k,j} - V \sum_{k,j} (\exp(i2\pi(\phi/\phi_0)s/L_x) a_{k,j+1}^\dagger a_{k,j} + a_{k+1,j}^\dagger a_{k,j} + h.c) + H_{int}, \quad (5)$$

where $\epsilon_{k,j}$ is the energy of a site (k, j) , chosen randomly between $-W/2$ and $W/2$ with uniform probability, V is a constant hopping matrix element, and s is the lattice unit. The interaction Hamiltonian is given by:

$$H_{int} = U \sum_{k,j>l,p} \frac{a_{k,j}^\dagger a_{k,j} a_{l,p}^\dagger a_{l,p}}{|\vec{r}_{k,j} - \vec{r}_{l,p}|/s} \quad (6)$$

where $U = e^2/s$. The distance $|\vec{r}_{k,j} - \vec{r}_{l,p}|/s = (\min\{(k-l)^2, (L_x/s - (k-l))^2\} + (j-p)^2)^{1/2}$. The interaction term represents Coulomb interaction between electrons confined to a 2D cylinder embedded in a 3D space.

We consider a 4×6 cylinder with $M = 24$ sites and $N = 3$ or $N = 4$ electrons. The size of the many body Hilbert space is $m = \binom{M}{N}$. The $m \times m$ Hamiltonian matrix is numerically diagonalized and the ground-state eigenvectors Ψ_N are obtained. The strength of e-e interactions, U , is varied between $0 - 22V$. The disorder strength is set to $W = 3V$

in order to assure RMT behavior for the non-interacting case. For each value of U , the results are averaged over 500 different realizations of disorder. The left lead is attached to the (1, 1) site and the right one to the (4, 6) site (point contacts). Assuming a fixed barrier transmission $|t_{L(R)}|^2 \equiv 1$, $\Gamma_{L(R)}$ is calculated using Eq. (4).

The average tunneling rate, its root mean square value, and the normalized fluctuations are presented in Fig 1 as function of the interaction strength U for $B = 0$ and $B \neq 0$. Using the expression of $\Gamma_{L(R)}$ in terms of Green functions given by Zyuzin and Spivak [16] one can use a diagrammatic summation similar to the one used in Fig. 1(a) of Ref. [18] for $\partial N/\partial\mu$ to obtain the RPA predictions for $\langle\Gamma_{L(R)}(U)\rangle$. For small values of U the RPA prediction [17] $\langle\Gamma_{L(R)}(0)\rangle/\langle\Gamma_{L(R)}(U)\rangle \sim 1 + (\kappa L/\pi) \sim 1 + 0.34U$ (where $\kappa = S_d\nu e^2$, $S_d = 2\pi$ for an infinite system and 2.50 for a 4×6 lattice [18], $\nu = (\Delta L^2)^{-1}$ and $L^2 = L_x L_y$) is followed for both $B = 0$ and $B \neq 0$. For larger values of U , the tunneling rate, $\Gamma_{L(R)}(U)$ is reduced below the RPA value. It is expected that the short range order induced by the interactions indeed reduce the overlap between $a^\dagger|\Psi_N\rangle$ and $|\Psi_{N+1}\rangle$.

The fluctuations in the tunneling rate depicted in Fig. 1b, also decrease as function of the interaction strength. In the RPA regime the fluctuations may be calculated by a diagrammatic expansion similar to the one used in Fig. 1(b) of Ref. [18] for $\delta^2\partial N/\partial\mu$. As in Refs. [16,18] a cutoff must be used in order to avoid divergences in the non-interacting case, but the influence of the interaction is cutoff independent resulting in $(\langle\delta^2\Gamma_{L(R)}(0)\rangle/\langle\delta^2\Gamma_{L(R)}(U)\rangle)^{1/2} = (1 + (\kappa L/\pi))^2$, while for stronger interactions $\langle\delta^2\Gamma_{L(R)}(U)\rangle$ is strongly suppressed.

To make connection to real samples we use the ratio between the average inter-particle Coulomb interaction and the Fermi energy $r_s = 1/\sqrt{\pi n}a_B$ (where n is the electronic density and a_B is the Bohr radius) corresponding to $r_s \sim \sqrt{\pi/6}(U/2V)$ for $N = 4$, $M = 24$. For all the above quantities there is a clear borderline around $U = 2 - 4V$ or $r_s \sim 1$. At low r_s values (high densities) the tunneling rate agrees well with RPA calculations, while for stronger interactions the results are qualitatively different. Identical behavior was observed for conductance peak spacings fluctuations discussed in Ref. [3]. In both cases the appearance

of short range correlations at $r_s \geq 1$ lead to a failure of RPA. It is important to bear in mind that for real systems the density in the leads $n = 2 - 3.5 \times 10^{11} \text{ cm}^{-2}$, while in the dot the density is probably lower, thus in all experimental systems $r_s \sim 1 - 2$. They hence correspond to a regime where RPA no longer holds.

The full probability distribution of the dimensionless parameter α for different values of the interaction is shown in Fig. 2. For $B = 0$ the distributions for all values of U are reasonably close to the RMT prediction, Eq. (1). Thus, moderate changes in the second moment of $\Gamma_{L(R)}$ (Fig. 1c) hardly influence the distribution of α . This is in good agreement with the experimental data [6,7]. Also in the presence of a magnetic field, the interaction strength has no major effect. Nevertheless, it is interesting to note that the dip at small values of α predicted by RMT (Eq. (1)) seen for small values of U (actually the dip is even larger than predicted which is an artifact of the small system size) disappears for higher values of U . The disappearance of the dip at higher interaction values is the result of the reduced sensitivity of the system to the magnetic field.

The auto-correlation, Eq. (2), between the height of the i -th peak at different values of magnetic fields is given according to RMT [12,13] for the GUE ensemble by

$$C(\phi, \Delta\phi) = \left[1 + \left(\frac{\Delta\phi}{\phi_c} \right)^2 \right]^{-2}. \quad (7)$$

Here $\phi = BA$ is the magnetic flux through a dot of area A , and $\phi_c = \phi_0/\sqrt{g\mathcal{K}}$, where for a diffusive dot g is the dimensionless conductance $g = E_c/\Delta$ (E_c is the Thouless energy) and $\mathcal{K} = 1$. For a ballistic dot, $E_c = v_F/\sqrt{A}$, and the geometrical factor, for the case of a flux line threading the dot \mathcal{K} is of order of unity [13]. This corresponds to $\phi_c \sim 0.1\phi_0$ for the experimental setup in Ref. [6], while the experimental value is $\phi_c \sim \phi_0$ [19]. Recently Alhassid [20] pointed out that for a uniform magnetic field in the dot \mathcal{K} is much smaller corresponding to $\phi_c \sim 0.5\phi_0$, which is still significantly lower than the experimental value.

The numerically calculated $C(\phi, \Delta\phi)$ for $\phi = 0.2\phi_0$ for different values of U is presented in Fig 3. Eq. (7) describes the $U = 0$ behavior for small values of $\Delta\phi$ quite well. From the value of $\phi_c = 0.5\phi_0$ one obtains $g = 4$ which is reasonable.

Since the ϕ dependent part in the diagrammatic calculation (which is similar to the calculation of the fluctuations [16,18]) does not depend on the interaction, one expects to first order no changes in Eq. (7) from RMT calculations [17]. Indeed, for small U there is only a weak influence of the interaction strength on the auto-correlation function. At larger values of $\Delta\phi$, $C(\phi, \Delta\phi)$ seems to be more sensitive to U and in the regime $V < U < 4V$ it is hard to fit it by a particular functional form. For $U > 4V$ ($r_s \geq \sqrt{2}$) the auto-correlation function can be fitted again to the functional form of Eq. (7) but the correlation flux is enhanced. For example, for $U = 10V$ $\phi_c = 1.75\phi_0$. Thus, although strong interactions reduce the average conductance peak height, they hardly influence the conductance distribution, again in agreement with both experiments [6,7]. The main effect is a reduced sensitivity to flux which is also manifested in the weak dependence of the conductance peak spacing fluctuations [4] on magnetic field. It is worth mentioning that tunneling experiments through excited states of heavily doped GaAs dots [9], where $r_s < 1$, find the RMT correlation flux. The numerical calculation gives the two particle correlation function as well [18]. It turns out that the point where the conductance starts to deviate from RPA ($r_s \sim 1$) is accompanied by the appearance of short range correlations. the same correlations that led to the large conductance peak spacing fluctuations are here responsible for the transmission statistics. We emphasize that Wigner crystallization occurs at much stronger interactions.

In conclusion, the appearance of short range electron correlations which are the result of the low electronic densities in the measured quantum dots explains not only the large conductance peak spacing fluctuations, but also other features pertaining to their transport properties. First, it has been numerically shown that although the conductance peak height distribution is well described by the single electron RMT, this distribution is valid also at the low density (strong interaction) regime. The strong interaction model also agrees with the results of the experiment in the presence of a magnetic field, which stands at odds with the predictions of the single electron RMT. Most importantly, the auto-correlation between peak heights exhibits an enhanced characteristic magnetic field for $r_s > 1$ which is in good agreement with the experimental observations [6,7,19]. Thus, the fact that the experiments

are performed at low densities ($r_s > 1$) for which the RPA theory no longer holds explains many of the puzzling features exhibited by these systems. A unified explanation for the large conductance peak spacing fluctuations and the peak height characteristics is hence provided. A better understanding of temperature effects, and a full analytic treatment of the short range correlations is still needed.

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FIGURES

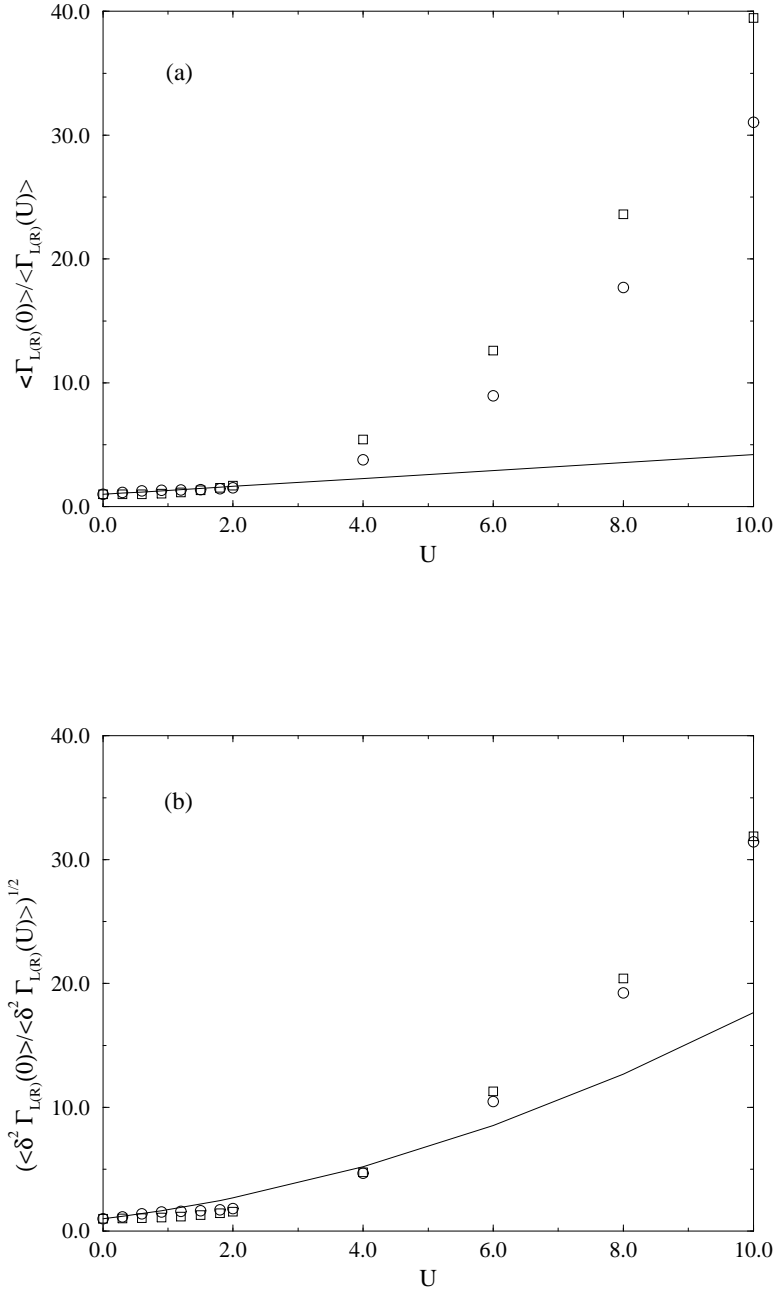


FIG. 1. The influence of the e-e interaction strength on the tunneling rate $\Gamma_{L(R)}$. Circles correspond to $\phi = 0$, while squares to $\phi = 0.4\phi_0$. The lines represent the prediction of an RPA theory. (a) The average tunneling rate, (b) the variance.

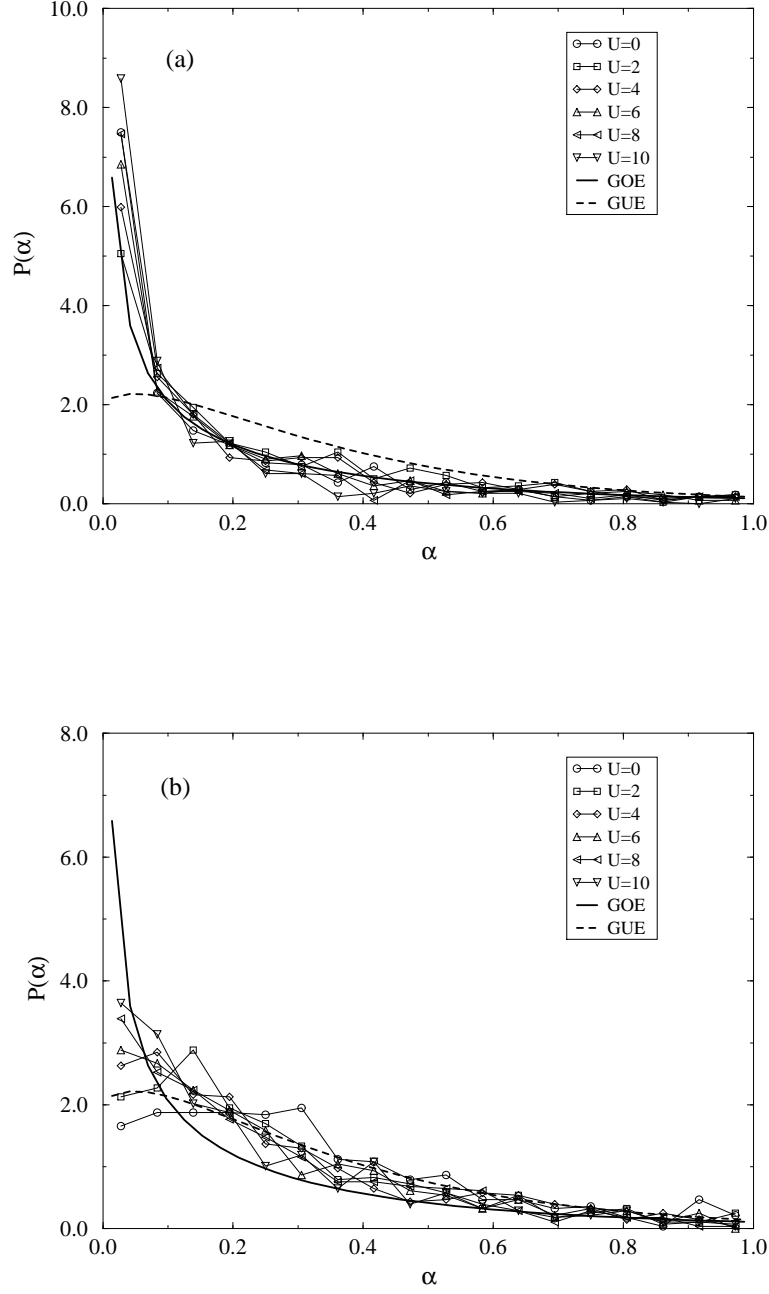


FIG. 2. The probability distribution of the dimensionless parameter α corresponding to the conductance peak height g_{max} for different values of the e-e interaction strength. (a) $\phi = 0$, (b) $\phi = 0.4\phi_0$. The lines correspond to the RMT predictions for $B = 0$ (GOE) and $B \neq 0$ (GUE).

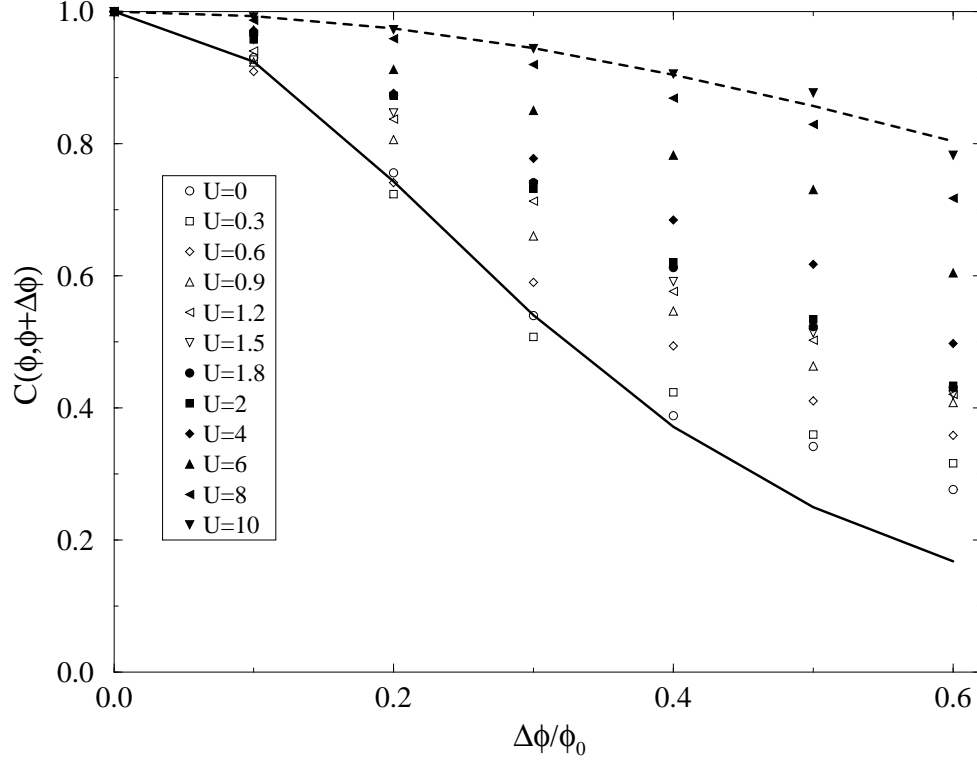


FIG. 3. The auto-correlation function for different values of the e-e interaction strength. The full line corresponds to Eq. (7) with $\phi_c = 0.5\phi_0$, while the dashed line to Eq. (7) with $\phi_c = 1.75\phi_0$.